More on Lemma 7

In my paper Bernoulli's Theorem , in Lemma 7, I claimed that given two fractions $\frac{A}{B}$ and $\frac{C}{D}$ then in the sequence of fractions $C_{\rm thon}$ in the sequence of fractic D $\frac{A}{B}$, $\frac{A+C}{B+D}$, $\frac{A+2C}{B-2D}$, $\frac{A+3C}{B-2D}$, ... in which each fraction is obtained B^3 $B+D^3$ $B+2D^3$ $B+3D^3$ More on Lemma 7

Ny paper <u>Bernoulli's Theorem</u>, in Lemma 7, I clain

n two fractions $\frac{A}{B}$ and $\frac{C}{D}$ then in the sequence of fi
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i $+C \quad A+2C \quad A+3C$ in w $+D$ ³ $B+2D$ ³ $B+3D$ ³ More on Lemma 7

Fr Bernoulli's Theorem , in Lemma 7, I claimed that

fractions $\frac{A}{B}$ and $\frac{C}{D}$ then in the sequence of fractions
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revious $+2C$ $A+3C$ in which $+2D$ ³ $B+3D$ ³ $2C \quad A+3C$ in which $2D$ ³ $B+3D$ ³ More on Lemma 7

oulli's Theorem , in Lemma 7, I claimed that
 $\frac{A}{B}$ and $\frac{C}{D}$ then in the sequence of fractions
 $\frac{A+3C}{B+3D}$, ... in which each fraction is obtained

fraction by adding C to the numerator and $+3C$ in which each free $+3D$ ³ $3C$ in which each from $3D$ ³

from the previous fraction by adding C to the numerator and D to the denominator, that each fraction in the sequence would be greater than the previous one if $\frac{A}{B} < \frac{C}{B}$, each fraction would be less than B D ³ C cook frequently would be less t D [,] \ldots is in the same of \ldots the previous one if $\frac{A}{B} > \frac{C}{D}$ and each fraction would be the same if C and each fraction would be the D $\frac{A}{B} = \frac{C}{D}$. B D \boldsymbol{C} D by adding C to the numerator and D to
fraction in the sequence would be greater
 $<\frac{C}{D}$, each fraction would be less than
id each fraction would be the same if
ive numbers then
 $\frac{A+C}{B+D} < \frac{C}{D}$ by adding C to the numerator and D to
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tive numbers then
 $\frac{A+C}{B+D} < \frac{C}{D}$

Here are the proofs:

Let A, B, C, and D be positive numbers then

1) If
$$
\frac{A}{B} < \frac{C}{D}
$$
 then $\frac{A}{B} < \frac{A+C}{B+D} < \frac{C}{D}$

proof

 $AD < BC$ $AD < BC$ $AB + AD < AB + BC$ $CD+AD < BC + CD$ $A(B+D) \leq B(A+C)$ $D(A+C) \leq C(B+D)$ So $A/B < (A+C)/(B+D)$ So $(A+C)/(B+D) < C/D$

$$
2 \\
$$

2
\n2) If
$$
\frac{A}{B} > \frac{C}{D}
$$
 then $\frac{A}{B} > \frac{A+C}{B+D} > \frac{C}{D}$
\nproof

proof

- $AD > BC$ $AD > BC$ $AB + AD > AB + BC$ CD + AD > BC +CD $A(B+D) > B(A+C)$ (A+C)D > (B + D)C So $A/B > (A+C)/(B+D)$ So $(A+C)/(B+D) > C/D$
- $\frac{A}{B} > \frac{A+C}{B+D} > \frac{C}{D}$

AD > BC

AD > BC

CD + AD > BC +CD

(A+C)D > (B + D)C

So (A+C)/(B+D) > C/D
 $\frac{A+C}{B+D} = \frac{A}{B}$ $\frac{A}{B} > \frac{A+C}{B+D} > \frac{C}{D}$

BC $CD + AD > BC + CD$
 $(A+C)D > (B+D)C$
 $B+D)$ So $(A+C)/(B+D) > C/D$
 $\frac{A+C}{B+D} = \frac{A}{B}$

3) If
$$
\frac{A}{B} = \frac{C}{D}
$$
 then $\frac{A+C}{B+D} = \frac{A}{B}$

proof

 $AD = BC$ $AB + AD = BC + AB$ $A(B + D) = B(A + C)$ So $A/B = (A + C)/(B + D)$

In lemma 7, I had $\frac{S}{S}$ in my formula and in part 2 of the proof $S-1$ and $\frac{1}{2}$ 1 contracts when $\frac{1}{2}$ I had $\frac{R}{R}$ in another formula. What happens if S = 1 or R = 1 ? $R-1$ and the second complete $R-1$

If $S = 1$ then $T = R + 1$ and $NT = NR+N$. This means there are no terms with a subscript greater than NR+N . If $R = 1$ then NR-N = 0. This means there are no terms with subscripts less than NR-N .

In either case, instead of calculating two N's and taking the higher

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of the two, you only need calculate one N and that N will give a ratio greater than C which will be correct.

If both R and S are 1 then T=2, $1/T = \frac{1}{2}$ and R/T = $\frac{1}{2}$. So the relative frequency range will be from 0 to 1 and the probability of that happening is 100%

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