More on Lemma 7

In my paper Bernoulli's Theorem , in Lemma 7, I claimed that given two fractions $\frac{A}{B}$ and $\frac{C}{D}$ then in the sequence of fractions $\frac{A}{B}, \frac{A+C}{B+D}, \frac{A+2C}{B+2D}, \frac{A+3C}{B+3D}, \ldots$ in which each fraction is obtained

from the previous fraction by adding C to the numerator and D to the denominator, that each fraction in the sequence would be greater than the previous one if $\frac{A}{B} < \frac{C}{D}$, each fraction would be less than the previous one if $\frac{A}{B} > \frac{C}{D}$ and each fraction would be the same if $\frac{A}{B} = \frac{C}{D}$.

Here are the proofs:

Let A, B, C, and D be positive numbers then

1) If $\frac{A}{B} < \frac{C}{D}$ then $\frac{A}{B} < \frac{A+C}{B+D} < \frac{C}{D}$

proof

AD < BC	AD < BC
AB +AD < AB+BC	CD+AD < BC + CD
A(B+D) <B(A+C)	D(A+C) < C(B+D)
So A/B < (A+C)/(B+D)	So (A+C)/(B+D) < C/D

2) If $\frac{A}{B} > \frac{C}{D}$ then $\frac{A}{B} > \frac{A+C}{B+D} > \frac{C}{D}$

proof

AD > BC
AB + AD > AB + BC
A(B+D) > B(A+C)
So A/B > (A+C)/(B+D)

AD > BC
CD + AD > BC +CD
(A+C)D > (B + D)C
So (A+C)/(B+D) > C/D

3) If $\frac{A}{B} = \frac{C}{D}$ then $\frac{A+C}{B+D} = \frac{A}{B}$

proof

AD = BC
AB + AD = BC + AB
A(B + D) = B(A + C)
So A/B = (A + C)/(B + D)

In lemma 7, I had $\frac{S}{S-1}$ in my formula and in part 2 of the proof I had $\frac{R}{R-1}$ in another formula. What happens if S = 1 or R = 1 ?

If S = 1 then T = R + 1 and NT = NR+N . This means there are no terms with a subscript greater than NR+N .
If R = 1 then NR-N = 0. This means there are no terms with subscripts less than NR-N .

In either case, instead of calculating two N's and taking the higher of the two, you only need calculate one N and that N will give a ratio greater than C which will be correct.

If both R and S are 1 then T=2, 1/T = ½ and R/T = ½ .
So the relative frequency range will be from 0 to 1 and the probability of that happening is 100%

Daniel Daniels